Fast Near-Optimal Energy Allocation for Multimedia Loading on Multicarrier Systems

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Abstract— **An efficient near-optimal energy allocation method for multimedia data transmitted using multicarrier modulation is investigated in this research. Our algorithm allocates energy to carriers by solving a simple set of linear equations, which is near-optimal for almost the entire SNR range. Furthermore, based on the proposed energy allocation algorithm, we develop asymptotic limits for the approximation error, the carrier energy, and the carrier rate. While limits for the special case with uniform noise and BPSK modulation were obtained before, our derivation applies to the more general case with non-uniform noise and higher-order M-QAM constellations. Finally, we link these limits with several types of multimedia and non-multimedia sources.**

Keywords— **Multimedia, QAM, QoS, UEP.**

I. Introduction

Multicarrier modulation [1] has received much attention due to its ability to adapt to environments with various channel conditions, e.g. narrowband interference, fading effects, impulse noise, etc. When these carriers are orthogonal to each other over the channel bandwidth, the technique is called Discrete Multitone Transmission (DMT) for wired channels and Orthogonal Frequency Division Multiplexing (OFDM) for wireless channels. The allocation of energy and/or rate to each carrier to improve the quality of service (QoS) is known as loading. In this research, we examine multimedia loading on multicarrier systems.

Early loading research was focused on DMT applications under channel capacity considerations. An acceptable margin was proposed in [2], [3] to offset the difficulty of transmitting signals near channel capacity. While this approach proves to be useful, it does not specifically take the symbol error probability, P_s, into account for higher order M-QAM (multilevel quadrature amplitude modulation) constellations. A data-loading scheme to reach uniform minimum P_s across carriers was developed in [4], [5]. More recently, a computationally efficient loading algorithm based on the SNR criterion was studied in [6]. While these approaches are useful for certain types of applications, they are not optimized for multimedia data transmission.

There has been some research on multimedia data transmission over multicarrier systems. Ho and Kahn [7] optimized the source coding scheme subject to channel conditions. Ho and Kahn [8] allocated power according to a source-channel distortion model, where the asymptotic limit for the BPSK modulation was derived. Zheng and Liu [9] proposed a bit rate allocation scheme to carriers, and demonstrated the performance gain for image and video transmission. Embedded modulation for a two-layer source was examined by Pradhan and Ramchandran [10].

Our current research differs from previous work in two aspects. First, we show that energy allocation can be obtained by solving a linear set of equations, rather than by a computationally intensive iterative process that is required by optimal multimedia allocation methods proposed previously. The near optimal results apply to almost the entire SNR range. Second, based on the proposed energy allocation algorithm, we develop a set of asymptotic limits for the approximation error, the carrier energy, and the carrier rate. While limits for the case with uniform noise and BPSK modulation were obtained before, our derivation applies to the more general case with non-uniform noise and higher-order M-QAM constellations. Finally, we link these limits with several types of multimedia and non-multimedia sources.

The remainder of this paper is organized as follows. Sec. II presents background on multicarrier allocations using M-QAM constellations for multimedia. Next, we present the linearized energy optimization algorithm in Sec. III. Asymptotic analysis of several bounds is given in Sec. IV. Simulation results are presented in Sec. V. Concluding remarks are given in Sec. VI.

II. Background on Multicarrier Allocation

A. Problem Formulation

The end-to-end data distortion D_T can be expressed as the sum of source distortion D_S and channel distortion D_C as

$$D_T = D_S + D_C. \qquad (1)$$

For a given source coding scheme, where D_S is fixed, minimizing D_T is equivalent to minimizing D_C. We begin with a multicarrier system composed of M independent carriers and a goal of minimizing D_C, subject to energy and rate constraints. The energies E_i, whose sum is E_T, and rates R_i, whose sum is R_T, with index $i \in \{1 \ldots M\}$, are distributed among these carriers. In the following, we consider a frequency selective fading channel, where each carrier has its own noise power spectral density, N_{0_i}.

The allocation scheme determines D_C, which can be written as

$$D_C = \sum_{i=1}^{Q} \sum_{j=1}^{Q} d_{ij} P(j|i) q_i \qquad (2)$$

where $d_{i,j}$ denotes the inter-symbol distortion between symbols i and j, $P(j|i)$ is the transitional probability that symbol i was sent and symbol j was received, q_i is the *a priori* probability of symbol i, and $Q = 2^{R_s}$ is the number of symbols, each of which contains R_s bits. With this definition, D_C is composed of elements related to source coding and transmission parameters. Source coding manifests itself through R_s, q_i, and d_{ij}. Transmission parameters manifest themselves through $P(j|i)$, which is dependent upon $\mathbf{e}$ and $\mathbf{r}$. Here, $\mathbf{e}$ and $\mathbf{r}$ are vectors of allocated energies and rates, respectively.

For large Q, the complexity involved in computing this distortion can be prohibitive. Hence, an accurate approximation is required to enable a feasible system design. A good approximation is given in [8]. It assumes that only single bit errors are likely to occur, which is valid for the SNR (E_b/N_0) range of interest in multimedia applications. This approximation can be formally stated via

$$P(j|i) \approx \begin{cases} 1 & N_b = 0 \\ p_{b_m} & N_b = 1 \\ 0 & N_b > 1 \end{cases} \quad (3)$$

where N_b is the number of bit errors between the transmitted and received symbols, and p_{b_m} denotes the probability of bit error at bit index m.

In a multicarrier system, the energy and rate for each can be modified while still yielding a constant average energy. For this case, $P(\cdot)$ is dependent upon the bit layer and the energy and rate allocated to it. Then, the distortion can be further approximated by

$$D_C \approx \frac{1}{k} \sum_{\substack{m=1 \\ i\in\{1\ldots M\}}}^{R_s} P_i(E_i; R_i) W_m \quad (4)$$

where W_m represents the weight associated with the source coding and inter-symbol distortions for bit layer m, k is the dimensionality of the source code ($k = 1$ for scalar), P_i, E_i, and R_i are the probabilities of error, energy and rate for carrier i, respectively. Although it may happen that R_s carriers are used (e.g. one bit per carrier), there are generally more bits than carriers. For this reason, each carrier requires higher order constellations, which will be discussed further in the next subsection.

B. M-QAM Constellations

There are many ways of constructing M-QAM constellations. They can be square, rectangular, circular, or a myriad of other shapes. Additionally, constellations may have regular or irregular forms. The most straightforward and mathematically tractable forms are square M-QAM constellations that result when two identical M-PAM constellations are superimposed on quadrature carriers. Although square constellations may not be optimal, they only suffer mild degradation relative to optimal constellations of a given order [11]. Note also that square M-QAM constellations form the basis for previous research with respect to multicarrier optimizations [4]- [9].

The symbol error probability for an $\mathrm{M}^{1/2}$-PAM constellation with energy E_{av}, is [11]

$$\hat{P}_{\sqrt{M}} = 2(1 - \frac{1}{\sqrt{2^R}}) \frac{1}{\sqrt{2\pi}} \int_{\sqrt{\frac{6E_{av}}{(2^R-1)N_0}}}^{\infty} e^{\frac{-z^2}{2}}\, \mathrm{dz}. \quad (5)$$

Similarly, P_M for square M-QAM constellations can be derived from (5), where one-half of the average energy is applied to each quadrature carrier. For all but the lowest E_b/N_0, the probability of error may be accurately approximated as

$$P_M \approx 4(1 - \frac{1}{\sqrt{2^R}}) Q\left(\sqrt{\frac{3E_{av}}{(2^R-1)N_0}}\right). \quad (6)$$

We will use the above approximation in all of our optimization results.

C. Lagrange Optimization

To find the optimum allocation using equality constraints for an objective function [12] that is continuous in energy and the rate, we can set up a Lagrangian cost function as

$$J = \frac{1}{k} \sum_{\substack{m=1 \\ i\in\{1\ldots M\}}}^{R_s} P_i(E_i; R_i) W_m + \lambda_1 \sum_{i=1}^{N} E_i + \lambda_2 \sum_{i=1}^{N} R_i \quad (7)$$

where λ_1 and λ_2 are Lagrange multipliers. The solution to this problem is obtained by taking derivatives with respect to E_i and R_i. However, since D_C is often a discrete function of the rate, derivatives can only be taken with respect to E_i[1]. Subsequently, algorithms such as Newton's method cannot be used to optimize energy and the rate jointly. Alternative rate allocation methods include full search, greedy search, etc. Full search may be feasible if the number of constellations is small. It was shown in [9] that their greedy algorithm has only slightly suboptimal performance as compared to the full search algorithm.

When the goal is to optimize energy for a fixed rate, the dimensionality of the problem is reduced. The new cost function becomes

$$J = \frac{1}{k} \sum_{\substack{m=1 \\ i\in\{1\ldots M\}}}^{R_s} P_i(E_i; R_i) W_m + \lambda \sum_{i=1}^{N} E_i. \quad (8)$$

The optimal solution can be found by differentiation, which results in a set of M independent non-linear equations in E_i:

$$\frac{\partial J}{\partial E_i} = \frac{1}{k} \frac{\partial P_i(E_i; R_i)}{\partial E_i} \widetilde{W}_i + \lambda = 0 \quad (9)$$

where $\widetilde{W}_i$ represents the partial sums of W_m that result from the mapping of bit layers to carriers. Iterative algorithms such as Newton's, bisection, and secant methods can be used to solve the nonlinear system of equations in (9). Specifically, they can be used to determine the optimal energy allocation for a given rate.

[1] The continuity requirement applies to the objective function, in this case D_C. For other objective functions such as those that optimize capacity or P_s, it is possible to differentiate with respect to the rate and obtain an suboptimal solution, where the suboptimal nature is due to the quantization of the rate to its nearest integer values

III. Proposed Energy Allocation Algorithm

Our new energy allocation algorithm for multimedia data loading is presented in this section. The significance of this work is twofold. First, by linearizing the energy search algorithm, we reduce the computational burden of iterative algorithms significantly. Additionally, there are convergence issues to be addressed in the iterative solution approach, which are not a factor under the new algorithm. Second, since our algorithm is highly accurate as compared to the optimal solution and because the allocation can be derived analytically, effects of optimal energy allocation for multimedia sources can be well explained. It would be difficult to have such insights using simulation results alone.

Since rate is discrete, it cannot be easily optimized using the method of Lagrange multipliers. A typical solution procedure is to focus on either the rate or energy allocation sequentially. For example, a rate allocation scheme is first chosen. Then, it is followed by an optimal energy allocation scheme. After that, the rate is updated based on the new energy allocation result. This process continues until the energy and rate allocation minimizes the overall distortion. We use the same process here and assume a rate allocation is given. Then, the Lagrangian optimization procedure can be used to optimize the energy. This can be written as

$$J = \frac{1}{k} \sum_{\substack{m=1 \\ i\in\{1\ldots M\}}}^{R_s} P_i(E_i) W_m + \lambda \sum_{i=1}^{M} E_i. \tag{10}$$

Next, the derivative of $J(\cdot)$ is taken with respect to E_i. Since all data will be transmitted via M-QAM, we take the derivative of (6) with respect to energy for each carrier that is used to transmit information. After taking the derivatives, we have M independent non-linear equations whose solution is the optimum energy allocation across all carriers. These equations are independent as a result of the simplifying assumptions made in (3). Each of these equations has the following form:

$$\lambda_{opt} = \frac{K_R}{2k} \widetilde{W}_i \left[e^{\frac{-3E_i}{(2^{R_i}-1)N_{0_i}}} \frac{3}{E_i(2^{R_i}-1)N_{0_i}} \right]^{\frac{1}{2}} \tag{11}$$

where K_R is the rate-dependent constant that multiplies the integral function in (6). Note that each carrier may have different energy, rate, and noise power which is denoted by i. Eq. (11) can be reduced to the form below.

$$\frac{e^{\alpha x}}{\beta x} = C \tag{12}$$

Equations of this type are called the exponential-linear equations, for which a closed-form solution is not possible. However, in the case of (11), we can make an approximation that proves to hold for a high degree of accuracy.

The optimum value of λ, denoted by λ_{opt}, is a constant for each independent equation of the form in (11). Therefore, setting two of the equations equal to each other and applying some elementary algebra, we can obtain the difference in carrier energy between carriers i and j as

$$\frac{E_i}{\eta_i} - \frac{E_j}{\eta_j} = -\frac{1}{3} \left\{ ln \left[\left(\frac{\widetilde{W}_j K_{R_j}}{\widetilde{W}_i K_{R_i}} \right)^2 \left(\frac{\eta_i}{\eta_j}\right) \right] + ln \left[\frac{E_i}{E_j} \right] \right\} \tag{13}$$

where

$$\eta_i = (2^{R_i} - 1) N_{0_i}. \tag{14}$$

We split the right hand side of (13) into two terms: one that is related to the source parameters, rate allocation, and noise power, and the other that is referred to as the "error" term. Specifically, we have

$$\beta_{ij} = -\frac{1}{3} ln \left[\left(\frac{\widetilde{W}_j K_{R_j}}{\widetilde{W}_i K_{R_i}} \right)^2 \left(\frac{\eta_i}{\eta_j}\right) \right] \tag{15}$$

and

$$\epsilon_{ij} = -\frac{1}{3} ln \left[\frac{E_i}{E_j} \right]. \tag{16}$$

The error term is a scaled value of the natural logarithm of the ratio of the optimal energy values for the two carriers. The performance of the algorithm is aided by the logarithm which lowers the carrier energy ratio. Although this error may be significant at low E_T, we will show that it approaches 0 with increasing energy for $\eta_i = \eta_j$. Then, we obtain

$$\lim_{\epsilon_{ij}\to 0} \left(\frac{E_i}{\eta_i} - \frac{E_j}{\eta_j} \right) = \beta_{ij} \tag{17}$$

where $\epsilon_{ij} \to 0$ as $E_i/E_j \to 1$. We will show that the effects of this term are negligible at moderate and even low E_b/N_0, and are highly dependent on source parameters. For cases with $\eta_i \neq \eta_j$ (e.g. non-uniform noise or rates), it can be shown that

$$\lim_{E_T\to\infty} \epsilon_{ij} = -\frac{1}{3} ln \left(\frac{1 + \eta_i/\eta_j}{1 + \eta_j/\eta_i} \right). \tag{18}$$

As shown in (15), β_{ij} results from three parameters, source coding, rate allocation between carriers, and noise power in each carrier. With respect to energy allocation, it is a constant. Therefore, we see that the resultant energy difference equations form a set of $(M-1)$ linear equations. When combined with the energy constraint

$$\sum_{i=1}^{M} E_i = E_T \tag{19}$$

we obtain a set of M linear equations in M unknowns which leads to a unique set of E_i. These equations can be expressed in matrix form as

$$\begin{pmatrix} \frac{1}{\eta_1} & \frac{1}{\eta_2} & 0 & \ldots & 0 \\ 0 & \frac{1}{\eta_2} & \frac{1}{\eta_3} & \ldots & 0 \\ \vdots & & \ddots & \ddots & \\ 0 & & \ldots & \frac{1}{\eta_{M-1}} & \frac{1}{\eta_M} \\ 1 & 1 & 1 & \ldots & 1 \end{pmatrix} \begin{pmatrix} E_1 \\ E_2 \\ \vdots \\ E_{M-1} \\ E_M \end{pmatrix} = \begin{pmatrix} \beta_{12} \\ \beta_{23} \\ \vdots \\ \beta_{(M-1)M} \\ E_T \end{pmatrix}. \tag{20}$$

Using vector and matrix notation with $\mathbf{A}$ as a square matrix, we can take the inverse of $\mathbf{A}$ to obtain the optimal energy allocation as

$$\mathbf{e} = \mathbf{A}^{-1}\mathbf{b}. \tag{21}$$

For highly distorted channels, the dimensionality of the problem can be reduced by simply removing those channels from the matrix equation in (21).

IV. Asymptotic Energy and Rate Analysis

A. Asymptotic Energy Analysis

It is important to understand the behavior of the carrier energy allocation as energy is increased. Based on (20), we gain insights into how and why energy is allocated in a specific manner. We will perform this analysis for the two carrier case below. This analysis extends easily to a larger number of carriers.

We begin by computing the optimal energy allocation for each carrier. This allocation is calculated from (21) for a given rate allocation. After some algebra, we can show that

$$\begin{aligned} E_1^* &= \frac{\eta_1 \eta_2}{\eta_1 + \eta_2}\left(\frac{E_T}{\eta_2} + \beta_1\right) \\ E_2^* &= \frac{\eta_1 \eta_2}{\eta_1 + \eta_2}\left(\frac{E_T}{\eta_1} - \beta_1\right) \end{aligned} \tag{22}$$

where $*$ denotes optimal energy allocation at a specific rate. From these equations, we see that the optimal energy is essentially a trade-off of the normalized E_T and the term driven by the rate allocation and source parameters, e.g. β_{12}. Also included in this term are the channel noise powers, which are fixed.

To understand how energy is distributed between carriers, we compute the ratio of these energies and take the limit as $E_T \to \infty$. This yields

$$\gamma \doteq \lim_{E_T \to \infty} \frac{E_1^*}{E_2^*} = \frac{(2^{R_1} - 1)N_{0_1}}{(2^{R_2} - 1)N_{0_2}} \tag{23}$$

where γ is the ratio of the carrier energies as $E_T \to \infty$. Then, at high SNR, the carrier energies become

$$\begin{aligned} E_1^\infty &= E_T \frac{\gamma}{1+\gamma} \\ E_2^\infty &= E_T \frac{1}{1+\gamma}. \end{aligned} \tag{24}$$

By examining (22)-(24), we see that, for large E_T, the effects of source parameters, specifically multimedia bit layering is reduced. As showed previously, the bit layer importance manifests itself via $\widetilde{W}_i$, of which β_{12} is a function. Since β_{12} is divided by E_T when computing the ratio of carrier energy, its importance is reduced as E_T increases.

However, the effect β_{12} has on the energy distribution is not only limited by E_T. From (15), we observe the effect of bit layering and rate on β_{12}. Specifically, for a given allocation, β_{12} will have a smaller effect as

$$\left(\frac{\widetilde{W}_j K_{R_j}}{\widetilde{W}_i K_{R_i}}\right)^2 \frac{\eta_i}{\eta_j} \to 1. \tag{25}$$

When this term goes to 1, γ is constant across E_b/N_0 and only becomes a function of the rate and noise power.

The rate at which the energy allocation scheme converges to its limiting case depends upon the type of data source that the allocation is applied to. It is also dependent upon the noise variation of the channel and rate allocation. For the case of uniform noise and rate, sources with a more uniform weight vectors converge more quickly than non-uniform ones, e.g. the Gaussian source that has a high degree of variability in the bit layer importance. This concept will be confirmed by simulation results in Sec. V.

B. Rate Analysis Under Asymptotic Energy

From the derivation of the carrier energy as $E_T \to \infty$, we can derive the asymptotic energy effects on the rate also. Using (24), P_s for each carrier can be calculated as

$$\begin{aligned} P_{s,1} &= 4\left(1 - \frac{1}{\sqrt{2^{R_1}}}\right) Q\left(\sqrt{\frac{3E_T}{(2^{R_1}-1)N_{0_1} + (2^{R_2}-1)N_{0_2}}}\right) \\ P_{s,2} &= 4\left(1 - \frac{1}{\sqrt{2^{R_2}}}\right) Q\left(\sqrt{\frac{3E_T}{(2^{R_1}-1)N_{0_1} + (2^{R_2}-1)N_{0_2}}}\right). \end{aligned} \tag{26}$$

We see that the there is a slight difference in these probabilities due to the scaling of the $Q(\cdot)$ function by the corresponding rate. In previous work, this scaling term has been left out. Here, we include it for completeness.

Now, we have symbol probabilities that are function of the rate only. We attempt to optimize the rate under a large value of E_T by setting up a new Lagrange optimization problem as

$$J = \frac{\widetilde{W}_1}{R_1} P_{s,1} + \frac{\widetilde{W}_2}{R_1} P_{s,1} + \lambda(R_1 + R_2). \tag{27}$$

After taking derivatives with respect to rate[2] and removing terms that have little significance, we obtain

$$R_1 - R_2 \approx log_2(N_{0_2}/N_{0_1}). \tag{28}$$

Eq. (28) implies that, unless there is a large noise power difference between the two channels, the optimal rate allocation is to split the rate evenly between the 2 carriers for an optimal energy allocation. This may seem intuitive, but it is not necessarily true for all energy allocation methods. For example, a uniform energy allocation may lead to a different result. Also note that, because of the requirement for even numbered bits only, it takes a large noise variation to cause a change from the uniform rate. We will show that this occurs for a carrier noise ratio (CNR) greater than ~ 5.2.

Subsequent to the methodology that we used to develop a the set of linear equations for energy allocation in (20), we can develop a similar set of linear equations using (28) for rate allocation at high E_b/N_0. Then the optimal rate allocation can be developed with limited computational requirements.

[2]Unlike earlier when we could not take the derivative with respect to the rate, terms that require derivative of partial sums drop out of the equation here.

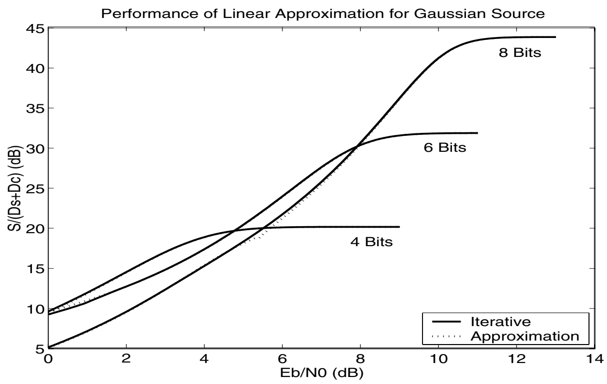

Fig. 1. The Signal to Total Distortion performance as a function of the SNR value (E_b/N_0) for two carriers with uniform noise.

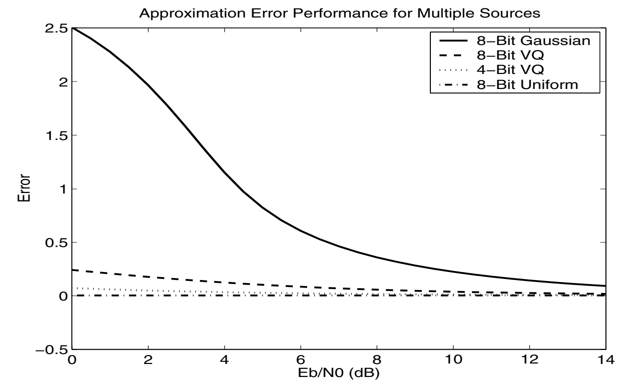

Fig. 2. Error performance over the E_b/N_0 range for multiple sources with uniform noise.

V. Simulation Results

Simulation results were generated from multiple sources to show performance characteristics across a range of potential data streams. These sources are meant to test the limits of the algorithm for layered and non-layered conditions. The first source considered is a typical scalar Gaussian source with $\sigma_s = 1$. The quantization levels were determined by the Lloyd-Max algorithm. This source tests the convergence properties of the solution algorithm since it provides the result of a high degree of variation in bit layer importance. For the 8-bit source, the bit layer importance is concentrated in the leading few bits, and the remaining bit importance drops off quickly. This effect is less severe for fewer bits. The next source is one with uniform bit layer, or non-layered, importance that is used as another extremum to demonstrate fast convergence. In the region between these two sources is a vector-quantized (VQ) Gaussian source, again with $\sigma_s = 1$. We obtained these weighting coefficients from [8]. Only 2-carrier results are demonstrated for clarity. The performance of the proposed near-optimal method against the iterative Newton method is evaluated using Matlab simulation. For a range of E_b/N_0, the speed for our algorithm was about 200 times faster than the Newton search.

We compare the performance of the proposed fast energy allocation method with that of the optimal iterative allocation in Fig. 1. Here the signal-to-distortion S/D_T, with D_T as shown in (1), is calculated as a function of E_b/N_0. Results for quantized Gaussian sources of length 4, 6, and 8 bits are given. We show the case of uniform noise since it is a tougher case for the fast algorithm, which can reach better results for non-uniform noise configurations, at low E_b/N_0. As shown in the figure, the fast algorithm generates results that are almost identical with the optimal ones over the entire range of E_b/N_0. Only the case of the 8-bit quantized source has a slight deviation from the optimal results in the E_b/N_0 range of 5-8 dB. We would expect performance to degrade below this point. It turns out that the fast algorithm generates negative energy results, which are not possible. We compensate for this fact by putting 1% of the carrier energy into carriers originally allocated negative energy. While not necessarily optimal, we observe that it provides a good rule of thumb. When the E_b/N_0 range is above 8 dB, the nearly optimal performance of the fast algorithm is observed. The effect of the error term (16) is negligible in this region.

Next, we examine the effects of the error term (16) in Fig. 2. As discussed earlier, these effects are added to the energy difference equation. For the case with uniform rate and noise, one half of the total gets added to the first carrier and subtracted from the second. For the non-uniform case, weighted versions that depend on the channel noise and rate allocation get added and subtracted. Our results verify that as energy increases for the uniform noise and rate case, the error term goes to zero. The largest error is shown for the 8-bit case at low E_b/N_0. We also present results for 4- and 6-bit VQ sources and the non-layered source. The error effect is significantly worse for sources with a large variation in bit importance, e.g. scalar Gaussian sources. Sources such as vector quantizes, and non-layered sources, exhibit less of an effect due the more constant nature of W_m.

The asymptotic energy analysis is verified in Fig. 3. Again, we use layered and non-layered sources as the limiting cases. Similar to the other results presented thus far, we demonstrate that, although both types of sources converge in the limit, they do so at different rates. Heavily layered sources such as the scalar source approach the asymptote at a much slower rate than the non-layered or VQ sources. By combining (22) and (23) and dividing by E_T, we see that the convergence performance is due to the magnitude of β_{12}. Smaller values of this term lead to faster convergence. Ideally, we prefer this term to be 0 for fastest convergence. This is precisely the case for a non-layered source with uniform noise and rate. Generally, this is true whenever (25) is valid.

Our asymptotic rate analysis is verified by data in Table I, where we compare the rate deviation, $R_1 - R_2$, for the 8-bit scalar quantized source and the 8-bit VQ source, with our limit for two different noise conditions. We define the CNR to be the ratio of carrier 2 to carrier 1. The first case is for uniform noise. We expect no rate deviation at high E_b/N_0. The scalar scalar is optimized, $R_1 = 2$ and

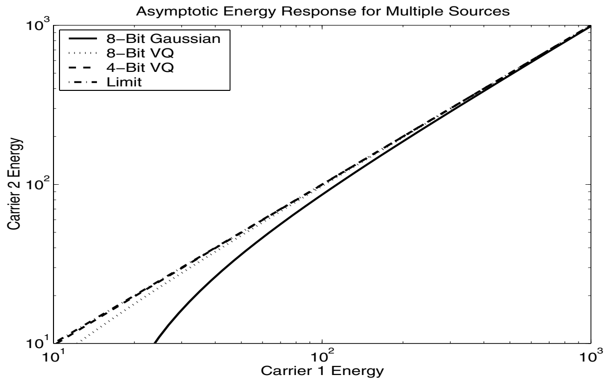

Fig. 3. Asymptotic energy response of multiple sources for two carriers with equal noise and bit rate.

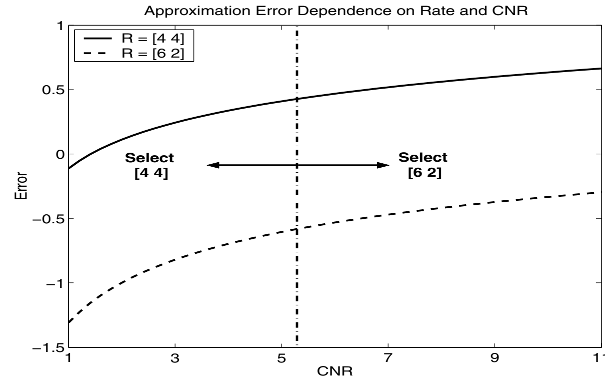

Fig. 4. Carrier noise ratio effect on approximation error for 8-bit Gaussian source at $E_b/N_0 = 13$ dB.

$R_2 = 6$, at low E_b/N_0 and reaches the predicted limit at $E_b/N_0 = 3$ dB. The VQ source maintains zero deviation from the predicted limit due to it low value of β_{12}. The case is similar for an CNR of 10. Here, the VQ source quickly converges to the optimal quantized deviation of 4, $R_1 = 6$ and $R_2 = 2$, whereas the scalar source requires an E_b/N_0 of 8 dB to reach the limit, once again, due to its large value of β_{12}.

In order determine the effects of non-uniform channels on approximation error, we calculate (16) with varying noise configurations. In Fig. 4, we illustrate the effects of CNR on approximation error for a scalar 8-bit source using the optimal energy allocation, where the error is exact and function of (15). Two curves for different rate configurations for $E_b/N_0 = 13$ dB are given. The optimal rate allocation changes at $CNR \approx 5.2$. At this point, the error transitions from the top curve to the bottom curve. This is due to the fact that, in accordance with (28), higher CNR leads to a rate allocation change, which in turn, has the effect of equalizing η_1 and η_2. Subsequently, there will be a reduction in the error as shown by (18). The rate allocation transition not only causes a decrease in the overall distortion, but also leads to a reduction in the approximation error.

TABLE I

Differential rate results for optimal search comparison with the calculated limit.

		E_b/N_0				
		0	1	2	3	≥ 8
$CNR = 1$	*scalar*	-4	-4	-4	0	0
	VQ	0	0	0	0	0
	predicted	0	0	0	0	0
$CNR = 10$	*scalar*	0	0	0	0	4
	VQ	4	4	4	4	4
	predicted	3.3	3.3	3.3	3.3	3.3

VI. Conclusion

An energy allocation algorithm that yields near optimal results across a wide range of E_b/N_0 was presented. The algorithm creates a set of linear equations whose solution is sufficiently close to the optimal energy allocation. The computational complexity is greatly reduced over iterative methods to compute the allocation. Subsequently, we provide an analysis of multimedia allocation using M-QAM via asymptotic energy and rate results and explain convergence properties of different sources. Our future work will include multiple sources with varying QoS requirements.

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