Precoded Multiuser OFDM Transceiver in Timing Asynchronous Environment

Shang-Ho Tsai, Yuan-Pei Lin, and C.-C. Jay Kuo

***Abstract*—The performance of a multiuser orthogonal frequency-division multiplexing (OFDM) transceiver, called the precoded multiuser OFDM (PMU-OFDM), with time asynchronous access is investigated in this paper. It is shown that multiaccess interference (MAI) due to time asynchronism is asymptotically zero as the number of parallel input symbols N becomes large in the PMU-OFDM system. Then, we show that PMU-OFDM with even or odd Hadamard–Walsh codewords can significantly suppress the MAI effect due to time asynchronism. For the half-loaded case, no sophisticated signal processing technique is needed by PMU-OFDM for MAI suppression. For the fully loaded situation, PMU-OFDM demands less complexity than does conventional OFDMA for interference suppression since PMU-OFDM deals with only one half of interferers.**

***Index Terms*—Code selection, multiaccess interference (MAI) free, multiuser detection, multiuser orthogonal frequency-division multiplexing (OFDM), orthogonal frequency-division multiple access (OFDMA), precoded multiuser orthogonal frequency-division multiple (PMU-OFDM), time offset.**

I. Introduction

MULTIUSER orthogonal frequency-division multiplexing (OFDM) systems such as multicarrier-code-division-multiple access (MC-CDMA) and orthogonal frequency-division multiple access (OFDMA) have attracted a lot of attention recently. The OFDMA system is multiaccess interference (MAI) free if time and frequency of all users are well synchronized. However, it is sensitive to the carrier frequency offset (CFO), which will cause significant MAI. Moreover, in uplink transmission, it is difficult to guarantee that all users are well aligned in time at the receiver, which leads to MAI as well. To overcome problems associated with time and frequency offsets, the use of signal processing techniques to estimate and compensate these offsets have been studied in recent years [1], [3]. Due to MAI, time and frequency offsets cannot be compensated in the receiver. Consequently, once these offsets are estimated in the receiver, a feedback mechanism is needed to send back the information so that every user can compensate these offsets at the transmitter end. However, multiuser estimation and/or information feedback demand extra complexity.

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S.-H. Tsai and Y.-P. Lin are with the Department of Electrical and Control Engineering, National Chiao Tung University, Hsinchu 300, Taiwan, R.O.C. (e-mail:shanghot@mail.nctu.edu.tw; ypl@mail.nctu.edu.tw).

C.-C. J. Kuo is with Ming Hsieh Department of Electrical Engineering, University of Southern California, Los Angeles, CA 90089-2564 USA (e-mail: cckuo@sipi.usc.edu).

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An approximately MAI-free multiaccess OFDM transceiver, called the precoded multiuser OFDM (PMU-OFDM), was proposed in [4]. In this system, each user transmits N parallel symbols. Each of the N symbols is precoded by an orthogonal code in the frequency domain to achieve multiaccess communication. When N is sufficiently large, the correlation between the precoded subchannel increases, and each symbol experiences nearly flat fading, which leads to approximately MAI-free. It is also shown in [4] that PMU-OFDM is nearly MAI free in the presence of CFO when even or odd Hadamard–Walsh codewords are used.

In this paper, we study PMU-OFDM under a time offset environment and obtain several interesting results. First, we show that PMU-OFDM with Hadamard–Walsh codes is approximately MAI free against time asynchronism. Second, we prove that the MAI power due to the time offset decreases in $O(1/N^2)$ if all M Hadamard–Walsh codewords are used (fully loaded system). For a half-loaded system, the MAI power can decrease even faster at a rate of $O(1/N^4)$ if a proper code is used. An intuitive explanation of this result is given as follows. With even or odd Hadamard–Walsh codewords, the codeword product is symmetric. When N is sufficiently large, the fading of precoded subchannels is almost monotonically increasing or decreasing (highly correlated), which can be regarded as an antisymmetric function. Since the summation of the codeword product and the monotonic fading is nearly zero, it leads to significant MAI suppression. Finally, we show that PMU-OFDM with proper code selection has a significantly lower MAI than that of OFDMA. Since the same codeword selection scheme can mitigate MAI due to CFO as well, we conclude that PMU-OFDM with even or odd Hadamard–Walsh codewords is robust against time and frequency asynchronism.

II. Time Asynchronism Analysis

For the system model of the PMU-OFDM, its properties and notations, we refer to [4]. Let the time offset of user i be τ_i, and there is no time offset for user j. For convenience, we define the codeword product as $r_{i,j}[m] = w_i[m]w_j[m]$, and let $\phi_{i,j}(\tau_i) = \sum_{m=0}^{M-1} r_{i,j}[m]e^{j(2\pi/NM)m\tau_i}$. The MAI from user i to user j due to time offset can be approximated by

$$\mathrm{MAI}_{j\leftarrow i}[k] \approx \frac{1}{M}\widetilde{\lambda}_i[k]x_i[k]\, e^{j(2\pi/NM)k\tau_i}\,\phi_{i,j}(\tau_i) \qquad (1)$$

where $\tilde{\lambda}_i[k]$ is the averaged channel gain and $x_i[k]$ is the transmitted symbol at the kth subchannel. If the Hadamard–Walsh code is used, we argue that, as the number N of parallel input symbols increases, the MAI will be zero asymptotically as follows. Since the maximum value of m is $M-1$, the term $e^{-j(2\pi/NM)v\tau_i}$ in (1) is approximately 1 if $N \gg |\tau_i|$. This approximation becomes more accurate as N increases. Since $\sum_{m=0}^{M-1} r_{i,j}[m] = 0$ for $i \neq j$, $\text{MAI}_{j\leftarrow i}[k] \approx 0$ for sufficiently large N. Thus, for sufficiently large N, PMU-OFDM is approximately MAI free even in a time offset environment.

III. Code Design for MAI Mitigation

We describe a code design to further suppress $\text{MAI}_{j\leftarrow i}[k]$. By using Taylor series expansion to the second order and $\sum_{m=0}^{M-1} r_{i,j}[m] = 0$ for $i \neq j$, we can rewrite $\phi_{i,j}(\tau_i)$ in (1) as

$$\phi_{i,j}(\tau_i) \approx \sum_{m=0}^{M-1} r_{i,j}[m]\left[-\frac{1}{2!}\left(\frac{2\pi}{NM}m\tau_i\right)^2 + j\frac{2\pi}{NM}m\tau_i\right] \triangleq \widetilde{\phi}_{i,j}(\tau_i). \tag{2}$$

We would like to evaluate $E\{|\text{MAI}_{j\leftarrow i}[k]|^2\}$, which is the averaged MAI power. It is assumed that transmitted symbols $x_i[k]$ and channel taps $h_i(n)$ are uncorrelated, $E\{x_i[k]x_i^*[k']\} = 0$ for $k \neq k'$, and $E\{h_i(n)h_i^*(n')\} = 0$ for $n \neq n'$. Also, let $\sigma_{x_i}^2 = E\{|x_i[k]|^2\}$ and $\sigma_{h_i}^2 = E\{|h_i(n)|^2\}$. Then, we have the following lemma.

Lemma 1: When all M Hadamard–Walsh codewords are used, i.e., in the fully loaded case, the maximum value of $E\{|\text{MAI}_{j\leftarrow i}[k]|^2\}$ can be approximated by

$$\max_{i,j} E\{|\text{MAI}_{j\leftarrow i}[k]|^2\} \approx L\sigma_{x_i}^2\sigma_{h_i}^2\left[\left(\frac{\tau_i}{N}\right)^2\frac{\pi^2}{4}\left(1-\frac{1}{M}\right)^2 + \left(\frac{\tau_i}{N}\right)^4\frac{\pi^4}{4}\left(1-\frac{1}{M}\right)^2\right] \tag{3}$$

and the maximum value occurs when $r_{i,j}[m]$ satisfies the following condition:

$$r_{i,j}[m] = \begin{cases} +1, & 0 \le m \le M/2-1 \\ -1, & M/2 \le m \le M-1. \end{cases} \tag{4}$$

Proof: Since $\widetilde{\lambda}_i[k] = \sum_{n=0}^{L-1} h_i(n)e^{-j(2\pi/N)nk}$, we have $E\{|\text{MAI}_{j\leftarrow i}[k]|^2\} \approx (L/M^2)\sigma_{x_i}^2\sigma_{h_i}^2|\widetilde{\phi}_{i,j}(\tau_i)|^2$. Hence, maximizing $E\{|\text{MAI}_{j\leftarrow i}[k]|^2\}$ is equivalent to maximizing $|\widetilde{\phi}_{i,j}(\tau_i)|^2$. From (2), $\widetilde{\phi}_{i,j}(\tau_i)$ can be rearranged as a real part plus an imaginary part as $\widetilde{\phi}_{i,j}(\tau_i) = \Re\{\widetilde{\phi}_{i,j}(\tau_i)\} + j\Im\{\widetilde{\phi}_{i,j}(\tau_i)\}$. Maximizing $|\widetilde{\phi}_{i,j}(\tau_i)|^2$ is equivalent to maximizing both $|\sum_{m=0}^{M-1} r_{i,j}[m]m|$ and $|\sum_{m=0}^{M-1} r_{i,j}[m]m^2|$. According to [2], the product of two arbitrary distinct Hadamard–Walsh codes yields a non-all-one Hadamard–Walsh codeword. Each of the non-all-one Hadamard–Walsh codeword has an equal number of $+1$ and -1. Since m and m^2 are monotonically increasing functions for $m \ge 0$, $|\sum_{m=0}^{M-1} r_{i,j}[m]m|$ and $|\sum_{m=0}^{M-1} r_{i,j}[m]m^2|$ are maximized if $r_{i,j}[m]$ has the same sign for $0 \le m \le M/2-1$, which is the condition in (4). In this case, we have $|\sum_{m=0}^{M-1} r_{i,j}[m]m^2| = (1/4)M^3[1-(1/M)]$, and $|\sum_{m=0}^{M-1} r_{i,j}[m]m| = (1/4)M^2[1-(1/M)]$, which lead to the results in (3). ■

As shown in (3), the maximum value of $E\{|\text{MAI}_{j\leftarrow i}[k]|^2\}$ depends on two terms. One is proportional to $1/N^2$ while the other is proportional to $1/N^4$. When N grows, the term proportional to $1/N^2$ dominates. Hence, the maximum value of $E\{|\text{MAI}_{j\leftarrow i}[k]|^2\}$ decreases at a rate proportional to $1/N^2$ asymptotically, which is contributed by $\Im\{\widetilde{\phi}_{i,j}(\tau_i)\}$. Hence, if we find a code such that $\Im\{\widetilde{\phi}_{i,j}(\tau_i)\} = 0$, the MAI due to the time offset will decrease at a rate of $O(1/N^4)$. This goal can be achieved by selecting a subset of codewords from the complete set of Hadamard–Walsh codes properly.

Lemma 2: The even/odd Hadamard–Walsh codewords satisfy the following properties

$$\sum_{m=0}^{M/2-1} r_{i,j}[m] = \sum_{m=M/2}^{M-1} r_{i,j}[m] = \begin{cases} M/2, & i = j \\ 0, & i \neq j \end{cases},$$

$$\sum_{m=0}^{M-1} r_{i,j}[m]m = 0. \tag{5}$$

Proof: We have $\sum_{m=0}^{M-1} r_{i,j}[m] = \sum_{u=0}^{M/2-1} r_{i,j}[u] + \sum_{v=M/2}^{M-1} r_i[v]$. For even (or odd) Hadamard–Walsh codewords, the second summation term is equal to $\sum_{v=M/2}^{M-1} r_{i,j}[v] = \sum_{v=M/2}^{M-1} r_{i,j}[M-1-v] = \sum_{v'=M/2-1}^{0} r_{i,j}[v']$. Since $\sum_{m=0}^{M-1} r_{i,j}[m] = 0$ for $i \neq j$, we get the first result in (5). Next, the term $\sum_{m=0}^{M-1} r_{i,j}[m]m$ can be decomposed into two terms as $\sum_{m=0}^{M-1} r_{i,j}[m]m = \sum_{u=0}^{M/2-1} r_{i,j}[u]u + \sum_{v=M/2}^{M-1} r_i[v]v$. Then, $\sum_{m=0}^{M-1} r_{i,j}[m]m$ can be manipulated as $\sum_{u=0}^{M/2-1} r_{i,j}[u]u + \sum_{u=0}^{M/2-1} r_{i,j}[M-1-u](M-1-u) = (M-1)\sum_{u=0}^{M/2-1} r_{i,j}[u] = 0$. ■

According to Lemma 2, we have $\Im\{\widetilde{\phi}_{i,j}(\tau_i)\} = 0$. Based on (1) and (2), we can apply similar arguments used in the proof of Lemma 1 to obtain the following theorem.

Theorem 1: Suppose that only even (or odd) Hadamard–Walsh codewords are used, the maximum value of $E\{|\text{MAI}_{j\leftarrow i}[k]|^2\}$ can be approximated by

$$\max_{i,j} E\{|\text{MAI}_{j\leftarrow i}[k]|^2\} \approx L\sigma_{x_i}^2\sigma_{h_i}^2\left[\left(\frac{\tau_i}{N}\right)^4\frac{\pi^4}{64}\right] \tag{6}$$

which occurs when $r_{i,j}[m]$ satisfies the following condition

$$r_{i,j}[m] = \begin{cases} +1, & 0 \le m \le M/4-1;\ M/2 \le m \le 3M/4-1 \\ -1, & M/4 \le m \le M/2-1;\ 3M/4 \le m \le M-1. \end{cases} \tag{7}$$

We see from (6) that, when the set of even (or odd) codewords is used as the spreading codes in the proposed PMU-OFDM system, the maximum value of $E\{|\text{MAI}_{j\leftarrow i}[k]|^2\}$ decreases at a rate of $O(1/N^4)$ as N becomes large. As compared with the general case where the MAI power decreases at a rate of $O(1/N^2)$, the use of even (or odd) codewords allows the system to reduce MAI at a much faster rate as N increases in a time asynchronous environment.

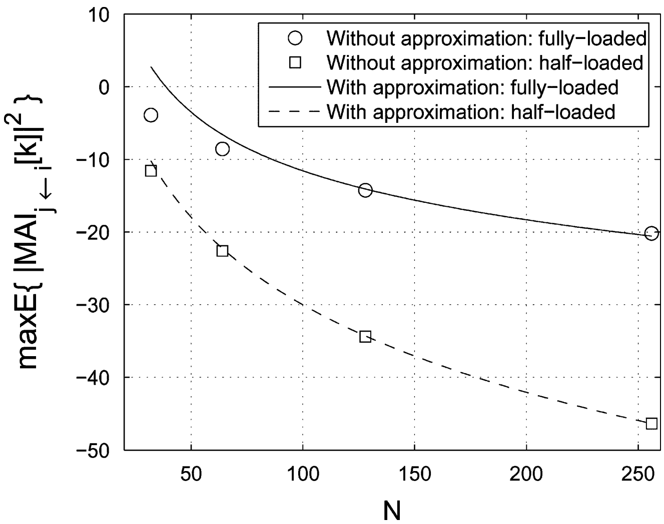

Fig. 1. Term $\max_{i,j} E\{|\mathrm{MAI}_{j\leftarrow i}[k]|^2\}$ with and without approximation as a function of N for the full- and half-load cases.

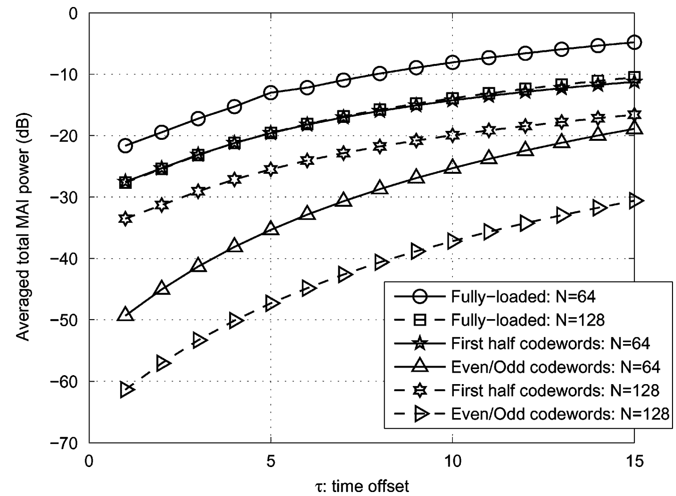

Fig. 2. MAI comparison for fully loaded and half-loaded cases with $N = 64$ and $N = 128$ (fully loaded: 16 users; half-loaded: 8 users).

IV. Simulation Results

A. Example 1: Rate of MAI Suppression

We verify that the second-order approximation in (2) holds approximately even when $|\tau_i|/N$ goes as large as $1/4$. We consider the flat fading channel case, where the approximation (1) becomes an equality. We compare the approximated maximum interference given in (3) of the fully loaded case (M users), (6) of the half-loaded case ($M/2$ users), and the exact formula, i.e., $E\{|\mathrm{MAI}_{j\leftarrow i}[k]|^2\} = (1/M^2)\sigma_{x_i}^2\sigma_{h_i}^2|\phi_{i,j}(\tau_i)|^2$. We set $M = 16$, $\sigma_{x_i}^2 = 1$, $\sigma_{h_i}^2 = 1$, and user i has a time offset $\tau_i = 16$. We consider the maximum MAI power from user i to user j, which occurs when $r_{i,j}[m]$ satisfies (4) for the fully loaded case, and when $r_{i,j}[m]$ satisfies (4) for the half-loaded case. The maximum MAI power with and without approximation as a function of N is shown in Fig. 1. The approximated maximum MAI power is plotted for all N from 32 to 256, while the exact maximum MAI power is obtained for $N = 32$, 64, 128 and 256. We see that the approximations are very accurate for both fully and half-load cases when $N \geq 64$. Since $\tau_i = 16$, the approximation appears to be accurate when $\tau_i/N \leq 1/4$. Moreover, we see that doubling N decreases the MAI power by 6 and 12 dB in the fully and half-load cases, respectively. These corroborate the results derived in Section III.

In Example 2, we consider the uplink case where each user has a different time offset and channel fading (no CFO effect is considered). Simulations are conducted with the following parameter setting: $M = 16$, BPSK modulation, multipath length $L = 10$, and independent identically distributed channel coefficients of complex Gaussian random variables with unit variance. Except for the target user who is assumed to have the correct timing, i.e., his/her time offset is zero, the time offsets of all other users are randomly assigned to be either $+\tau$ or $-\tau$. Other parameter settings for the simulation can be found in [4]. In Example 3, the parameter setting is the same as that in Example 2 except that $L = 10$, and the variance of the channel coefficient decays exponentially, namely, $\sigma_l^2 = e^{-\alpha l/L}$, where $\alpha = 2$ and $l = 0, 1, \ldots, L-1$.

B. Example 2: Comparison of MAI Suppression

We first consider the fully loaded case (16 users). The averaged total MAI are plotted as a function of the time offset for $N = 64$ and $N = 128$ as in Fig. 2. Moreover, as N increases from 64 to 128, the MAI decreases by around 6 dB, which corroborates the theoretical result that the MAI is proportional to $O(1/N^2)$ again. Next, we consider the half-loaded case (eight users) and compare two code selection methods: the use of only even or odd Hadamard–Walsh codewords and the use of the first $M/2$ codewords of M Hadamard–Walsh codes generated by the Kronecker product. Note that the MAI using even codewords and that using odd codewords are overlapping. For $N = 64$, as compared with the fully loaded case, the use of even or odd codewords can reduce the MAI by 14–29 dB, while the use of the first $M/2$ codewords only improve the MAI by around 5–6 dB. For $N = 128$, the use of even or odd codewords can reduce the MAI by 20–35 dB while the use of the first $M/2$ codewords again only improve the MAI by around 5–6 dB. We also see a 12-dB MAI power reduction as N increases from 64 to 128 for the proposed half-loaded case. Thus, the MAI decreases at a rate proportional to $1/N^4$ with the proposed code selection.

C. Example 3: Performance Comparison Between OFDMA and PMU-OFDM

We first evaluate the averaged total MAI by considering the MAI at the detection stage, i.e., after full equation (FEQ). The MAI performance of the two systems with $N = 128$ is shown in Fig. 3. In the fully loaded case, OFDMA has less MAI than PMU-OFDM. However, in the half-loaded case, PMU-OFDM has much smaller MAI than OFDMA by around 1–19 dB due to the use of the proposed code selection scheme. It is worthwhile to point out that both systems do not give good performance

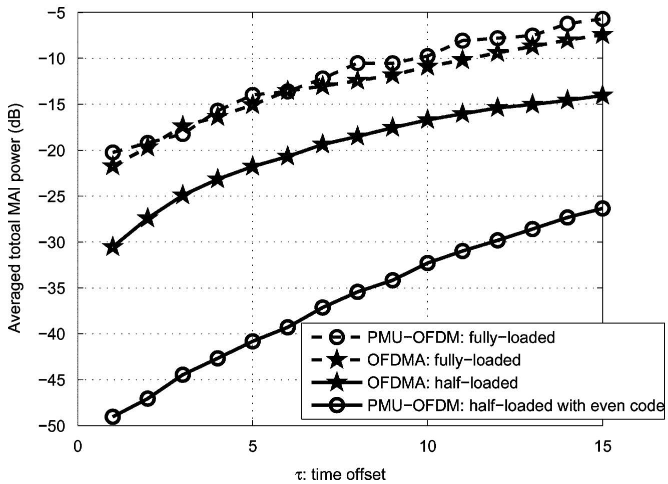

Fig. 3. Averaged total MAI comparison for PMU-OFDM and OFDMA in a frequency-selective channel with $L = 10$ (fully loaded: 16 users; half-loaded: 8 users).

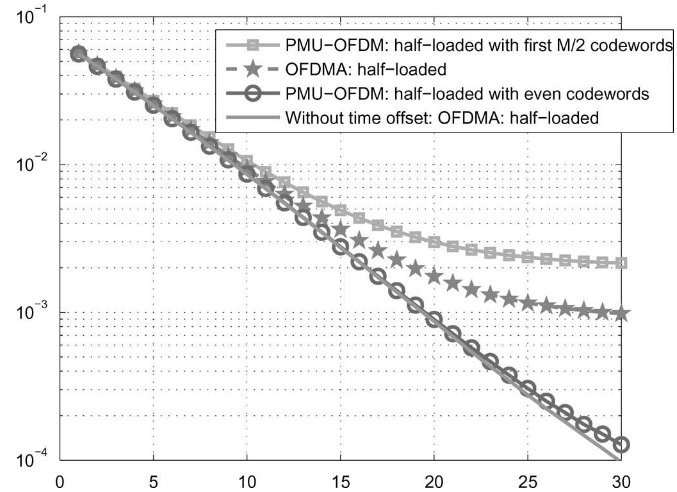

Fig. 4. BER performance comparison for PMU-OFDM and OFDMA in a frequency-selective channel with $L = 10$ and time offset level $|\tau_j| = 13$.

in the fully–loaded case. More sophisticated signal processing such as multiuser (MUD) can be used to suppress MAI. For PMU-OFDM, each user only has to deal with $M/2$ interferers while the other $M/2 - 1$ interferers has negligible MAI to this user. In contrast, each user has to deal with $M - 1$ interferers in OFDMA. In other words, PMU-OFDM demands less complexity for interference suppression than OFDMA in the fully loaded case. This is another important advantage of PMU-OFDM.

Assume that every user, except for the target user, has a time offset of $|\tau_i| = 13$, in the two systems. All users are chosen to be the target user in turn. We evaluate the bit error rate (BER) when there is no feedback, which is shown in Fig. 4. For comparison, we also show the curve for OFDMA without the time offset. We see that even in a serious timing mismatch environment as given in this example, PMU-OFDM with the proposed code scheme can still achieve comparable performance as that of OFDMA without a time offset. In contrast, the performance of OFDMA degrades significantly due to time asynchronism. The performance of PMU-OFDM with the first $M/2$ codewords is plotted as a benchmark. We see that its performance is close to that of the half-loaded OFDMA system, which is much worse than the proposed code scheme. Note that for a large SNR value, MAI becomes dominant so that an increased SNR value no longer improves the performance in this case.

V. Conclusion

The time asynchronism effect of the PMU-OFDM was studied. We showed analytically that the MAI power due to the time offset can be reduced by using Hadamard–Walsh codewords in PMU-OFDM. For a half-loaded system, we proposed a code selection scheme of using $M/2$ even or odd Hadamard–Walsh codewords, which yields an even better MAI suppression. Since PMU-OFDM does not require sophisticated MUD for multiuser communication, time offset compensation can be done at the receiver side, and we can have a new transceiver design with a much simpler time synchronization mechanism.

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